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Dispersion forces between an atom and a perfectly conducting wedge

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Abstract

We consider the interaction between an electrically polarizable atom in its fundamental state and a wedge constituted by two semi-infinite perfectly conducting plates. Using a formalism based on a master equation, we compute the dispersion force on the atom for both retarded and non-retarded regimes.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Dispersion forces are generally understood as those that occur between neutral objects that do not possess any permanent electric or magnetic moments, as the van der Waals force between two neutral but polarizable atoms or between a polarizable atom and a wall. These forces originate from the unavoidable quantum fluctuations, which are always present in nature and cannot be neglected under certain circumstances. The dominant contribution to the interaction of two neutral but polarizable objects comes from the dipole–dipole interaction, the only one we shall be concerned with in this paper. Two distinct distance regimes are worth studying, namely, the non-retarded regime (short distance regime) and the retarded one (large distance regime). The occurrence of a dominant transition wavelength naturally fixes a length scale which allows a characterization for these two regimes. Retardation effects become important as soon as typical distances between the two interacting objects are of the order of the dominant transition wavelength.

Since the seminal papers by London [1] (non-retarded force between two polarizable atoms), by Casimir and Polder [2] (retardation effects on the London–van der Waals force between two polarizable atoms and between an atom and a perfectly conducting plate) and by Lifshitz [3] (who developed a general theory of dispersive van der Waals forces between macroscopic dielectric bodies, valid also for $T \neq 0$), a wide knowledge about dispersion forces has been achieved: higher multiple moments and N-atoms interactions have been

considered; non-additivity of these forces have been studied; the oscillatory behaviour of the interaction between an excited atom and a wall has been obtained [4]; application of linear response theory has been done [5]; electric and magnetic interactions have been treated in equal footing [6]; atom–cavity interaction has been considered [7, 8]; thermal contribution to the Casimir–Polder force has been computed [9]; the influence of the Casimir–Polder force in Bose–Einstein condensates has been found [10, 11]; the importance of the Casimir–Polder force in carbon nanotubes has been recognized [12]; for the computation of the force on a neutral atom near some specific microstructures, see Eberlein and Zietal [13] (see also references therein); and many others. Nowadays, dispersion forces are important in many areas of science and find applications in quite unexpected situations varying from biology, chemistry and physics to engineering and nanotechnology. For a recent discussion on this subject from the point of view of macroscopic QED in linear media, which contains a detailed historical survey as well as a vast list of references, see [14] and references therein. The first experiment conceived to measure directly the atom–body interaction was done by Sukenik *et al* [15]. These authors analysed the deflection of ground-state sodium atoms crossing a micron-sized parallel-plate cavity (in fact, a wedge with a very small angle). The authors conclusively confirmed the existence of retardation effects. There are other experiments as, for example, the Orsay experiment, which measures the force between an atom and a dielectric wall by analysing the reflection of atoms by evanescent-wave atomic mirrors [16], and the Tokyo experiment, based on quantum reflection by the Casimir–Polder force [17] and even measurements that employ the influence of dispersion forces on BEC [18].

Our purpose is to obtain the dispersion force, in any distance regime, exerted on an electrically polarizable atom in ground state, which is near a wedge formed by two perfectly conducting plates of infinite extent. The atom–wedge system has been considered before by Brevik *et al* [19] but these authors computed the dispersion force on the atom only in the retarded regime. They based their calculations in a previous work on the Casimir effect for a perfectly conducting wedge [20] and the inclusion of a dielectric was done in [21]. Since the wedge geometry was employed in the first measurement of the Casimir–Polder force [15] and it may be convenient in future ones, we think it is of some relevance to generalize the results in [19] by providing the calculation of the dispersion force also in the non-retarded regime.

2. Dispersion potential for the atom–wedge system

Consider an atom and a perfectly conducting wedge as indicated in figure 1, where ρ is the distance between the atom and the corner, chosen as the $\mathcal{OZ}$ axis, and ϕ is the polar angle, measured with respect to the $\mathcal{OXZ}$ plane. We shall employ a method developed by Dalibard *et al* [22], based on a master equation to describe a small system interacting with a large one (referred to as a reservoir). In our case the small system will be the atom, while the large one will be the radiation field submitted to the appropriate boundary conditions imposed by the wedge. This approach provides general expressions for the level shifts and energy exchange rates of the system. Two contributions appear: one from the fluctuation of the reservoir, denoted by fr, and the other from the reaction of the reservoir, denoted by rr.

The level shifts δE_a for an atom in the state $|a\rangle$ interacting with the radiation field in the dipole approximation are given by [23]

$$\delta E_a = \delta E_a^{rr} + \delta E_a^{fr}, \tag{1}$$

$$\delta E_a^{rr} = -\frac{1}{2}\sum_j \sum_{\mathbf{k}\lambda} \alpha_{aj}^{\prime(-)}(k)\left|f_{\mathbf{k}\lambda}^j(\mathbf{x})\right|^2, \tag{2}$$

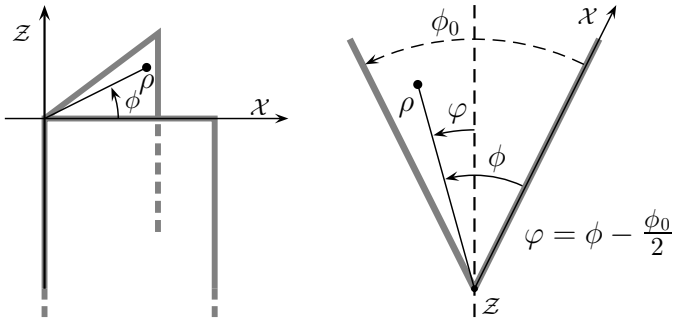

Figure 1. Atom–wedge system: the figure on the right shows a transverse cut.

$$\delta E_a^{fr} = -\sum_j \sum_{\mathbf{k}\lambda} \alpha_{aj}^{\prime(+)}(k) \left| f_{\mathbf{k}\lambda}^j(\mathbf{x}) \right|^2 \left(\langle n_{\mathbf{k}\lambda} \rangle + \frac{1}{2} \right), \tag{3}$$

$$\alpha_{aj}^{\prime(\mp)}(k) = \sum_b \frac{\alpha_{ab}^j k_{ba}}{2} \left[\mathcal{P} \frac{1}{k + k_{ba}} \pm \mathcal{P} \frac{1}{k - k_{ba}} \right], \tag{4}$$

with the electric field being $\mathbf{E}(\mathbf{x}, t) = \sum_{\mathbf{k}\lambda} \left(\mathbf{f}_{\mathbf{k}\lambda}(\mathbf{x})\, \mathrm{e}^{\imath\omega_k t} a_{\mathbf{k}\lambda}^\dagger + \mathbf{f}_{\mathbf{k}\lambda}^*(\mathbf{x})\, \mathrm{e}^{-\imath\omega_k t} a_{\mathbf{k}\lambda} \right)$, where $\omega_k = kc$, $[a_{\mathbf{k}\lambda}, a_{\mathbf{k}'\lambda'}] = \left[a_{\mathbf{k}\lambda}^\dagger, a_{\mathbf{k}'\lambda'}^\dagger \right] = 0$, $\left[a_{\mathbf{k}\lambda}, a_{\mathbf{k}'\lambda'}^\dagger \right] = \delta_{\mathbf{k}\mathbf{k}'} \delta_{\lambda\lambda'}$, $\mathcal{P}$ means the principal part, $k_{ab} = (E_a - E_b)/\hbar c$, so that $|k_{ab}|c$ is the transition frequency between states $|a\rangle$ and $|b\rangle$, $\langle n_{\mathbf{k}\lambda} \rangle$ is the average number of photons in the mode $\mathbf{k}\lambda$ and coefficients α_{ab}^j are defined as $\alpha_{ab}^j = -2|\langle a|d_j|b\rangle|^2/(\hbar c k_{ab})$, with d_j being the j-component of its dipole moment operator $\mathbf{d} = -e\mathbf{r} = -e(x_1, x_2, x_3) = (d_1, d_2, d_3)$. For convenience, we first consider the field modes of a wedge with a coaxial cylindrical shell of radius R which will be taken to infinity at the appropriate moment. The transverse electric (TE) and transverse magnetic (TM) modes of the quantized electric field for this setup are borrowed from Saharian's paper [24] and are given by

$$\mathbf{f}_{\mathbf{k},m,n}^{\mathrm{TM}}(\mathbf{x}) = \beta_{qm}(\gamma_{|m|,n} R) \left(\gamma_{|m|,n}^2 \hat{z} - \imath k_z \nabla_t \right) J_{q|m|}(\gamma_{|m|,n} \rho) \sin(q|m|\phi)\, \mathrm{e}^{-\imath(k_z z - \omega_k t)},$$

$$\mathbf{f}_{\mathbf{k},m,n}^{\mathrm{TE}}(\mathbf{x}) = \imath k \beta_{qm}(\eta_{|m|,n} R) \hat{z} \times \nabla_t [J_{q|m|}(\eta_{|m|,n} \rho) \cos(q|m|\phi)\, \mathrm{e}^{-\imath(k_z z - \omega_k t)}],$$

where m is an integer for TE modes and an integer different from zero for TM modes,

$$J_{q|m|}(\gamma_{|m|,n} R) = J'_{q|m|}(\eta_{|m|,n} R) = 0, \qquad X_\nu(x) = \left[J_\nu'^2(x) + \left(1 - \frac{\nu^2}{x^2} \right) J_\nu^2(x) \right]^{-1},$$

$$\beta_{qm}^2(x) = \frac{2q\hbar c}{\pi k} X_{qm}(x),$$

$$\nabla_t = \hat{\rho}\partial_\rho + \frac{1}{\rho}\hat{\phi}\partial_\phi, \qquad q = \pi/\phi_0, \qquad k^2 = \kappa_{mn\lambda}^2 + k_z^2,$$

$$\kappa_{mn1} = \gamma_{|m|,n} \quad \text{and} \quad \kappa_{mn2} = \eta_{|m|,n}.$$

The atom–boundary interaction is given by the position-dependent part of the energy shift induced by the atom–field interaction (with appropriate BC). Computing first the (rr) contribution, we have

$$\delta E_a^{rr} = -\frac{q\hbar c}{\pi} \int_{-\infty}^{\infty} \mathrm{d}k_z \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \sum_{\lambda,\sigma} \frac{\kappa_{mn\lambda}^4 X_{qm}(\kappa_{mn\lambda} R)}{\sqrt{\kappa_{mn\lambda}^2 + k_z^2}} \alpha_{a\sigma}^{\prime(-)} \left(\sqrt{\kappa_{mn\lambda}^2 + k_z^2} \right) Q_{qmn}^{\sigma,\lambda}(\rho, \phi),$$

where $\sigma = \rho, \phi, z$, the index $\lambda = 1, 2$ refers to the TM and TE modes, respectively, and

$$
\begin{aligned}
Q^{\rho,1}_{qmn}(\rho,\phi) &= \frac{k_z^2}{\kappa_{mn1}^2} J'^2_{qm}(\kappa_{mn1}\rho)\sin^2(qm\phi),\\
Q^{\phi,1}_{qmn}(\rho,\phi) &= \frac{k_z^2 q^2 m^2}{\kappa_{mn1}^4 \rho^2} J^2_{qm}(\kappa_{mn1}\rho)\cos^2(qm\phi),\\
Q^{z,1}_{qmn}(\rho,\phi) &= J^2_{qm}(\kappa_{mn1}\rho)\sin^2(qm\phi), \qquad Q^{z,2}_{qmn}(\rho,\phi) = 0,\\
Q^{\phi,2}_{qmn}(\rho,\phi) &= \left(1+\frac{k_z^2}{\kappa_{mn2}^2}\right) J'^2_{qm}(\kappa_{mn2}\rho)\cos^2(qm\phi),\\
Q^{\rho,2}_{qmn}(\rho,\phi) &= \left(1+\frac{k_z^2}{\kappa_{mn2}^2}\right)\frac{q^2m^2}{\kappa_{mn2}^2\rho^2} J^2_{qm}(\kappa_{mn2}\rho)\sin^2(qm\phi).
\end{aligned}
$$

Using the generalized Abel–Plana summation formula and taking $R \to \infty$, we get [24]

$$
\begin{aligned}
\delta E_a^{rr} &= -\frac{q\hbar c}{2\pi}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}k_z \int_0^{\infty} \frac{\kappa^3 \mathrm{d}\kappa}{\sqrt{k_z^2+\kappa^2}} \sum_{\sigma} \alpha'^{(-)}_{a\sigma}\left(\sqrt{k_z^2+\kappa^2}\right) S^{\sigma}_{\kappa qm}(\rho,\phi),\\
S^{z}_{\kappa qm}(\rho,\phi) &= J^2_{qm}(\kappa\rho)\sin^2(qm\phi),\\
S^{\phi}_{\kappa qm}(\rho,\phi) &= \left[\left(1+\frac{k_z^2}{\kappa^2}\right) J'^2_{qm}(\kappa\rho) + \frac{k_z^2 q^2 m^2}{\kappa^4\rho^2} J^2_{qm}(\kappa\rho)\right]\cos^2(qm\phi),\\
S^{\rho}_{\kappa qm}(\rho,\phi) &= \left[\frac{k_z^2}{\kappa^2} J'^2_{qm}(\kappa\rho) + \left(1+\frac{k_z^2}{\kappa^2}\right)\frac{q^2m^2}{\kappa^2\rho^2} J^2_{qm}(\kappa\rho)\right]\sin^2(qm\phi).
\end{aligned}
$$

There is no need for a cut-off function: the polarizability guarantees the convergence of the sums for the position-dependent part of $\delta E_a^{(rr)}$. It can be shown by explicit calculations that the (fr) contribution is obtained by $\alpha'^{(-)}_{a\sigma}(k) \longrightarrow \alpha'^{(+)}_{a\sigma}(k)(2\langle n_{\mathbf{k}\lambda}\rangle + 1)$. A further simplification is possible only for positive integer values of q ($q = 1, 2, 3, \ldots$). Using a generalization of the addition theorem for Bessel functions [25],

$$
\sum_{m=-\infty}^{\infty} J_{qm}(\kappa\rho) Z_{\nu+qm}(\kappa\rho)\,\mathrm{e}^{2\imath qm\phi} = \frac{1}{q}\sum_{l=0}^{q-1}(-1)^{\nu/2}\,\mathrm{e}^{-\imath\nu\psi_l} Z_{\nu}(2\kappa\rho\sin\psi_l),
$$

where Z_ν is a solution of Bessel's equation and $\psi_l = \phi + \vartheta_l$, $\vartheta_l = \pi l/q$, the sums over m can be evaluated. Passing to the spherical coordinates ($\kappa = k\sin\theta$, $k_z = k\cos\theta$) and making use of the identities involving Bessel functions, we get

$$
\begin{aligned}
\delta E^{rr}_{a,z} &= \frac{\hbar c}{2\pi}\sum_{l=0}^{q-1}\int_0^{\infty} \mathrm{d}k\, k^3 \alpha'^{(-)}_{az}(k)[G_{\parallel}(2k\rho\sin\psi_l) - G_{\parallel}(2k\rho\sin\vartheta_l)],\\
\delta E^{rr}_{a,\phi} &= \frac{\hbar c}{2\pi}\sum_{l=0}^{q-1}\int_0^{\infty} \mathrm{d}k\, k^3 \alpha'^{(-)}_{a\phi}(k)[H_{\phi}(2k\rho,\psi_l) + H_{\phi}(2k\rho,\vartheta_l)],\\
\delta E^{rr}_{a,\rho} &= \frac{\hbar c}{2\pi}\sum_{l=0}^{q-1}\int_0^{\infty} \mathrm{d}k\, k^3 \alpha'^{(-)}_{a\rho}(k)[H_{\rho}(2k\rho,\psi_l) - H_{\rho}(2k\rho,\vartheta_l)],
\end{aligned}
$$

where we defined

$$
\begin{aligned}
G_{\perp}(x) &= \frac{\cos x}{x^2} - \frac{\sin x}{x^3}; \qquad G_{\parallel}(x) = \frac{\sin x}{x} + \frac{\cos x}{x^2} - \frac{\sin x}{x^3},\\
H_{\phi}(x,\psi) &= G_{\parallel}(x\sin\psi)\sin^2\psi + 2G_{\perp}(x\sin\psi)\cos^2\psi,\\
H_{\rho}(x,\psi) &= G_{\parallel}(x\sin\psi)\cos^2\psi + 2G_{\perp}(x\sin\psi)\sin^2\psi.
\end{aligned}
$$

We still need to evaluate the k-integrals. From the expressions for $\alpha'_{a\sigma}(k)$, these integrals have the form

$$\mathcal{A}_\lambda^{(\pm)}(\omega, f) = \int_0^\infty \mathrm{d}k f(k) \left[\mathcal{P}\frac{1}{k+\omega} \mp \mathcal{P}\frac{1}{k-\omega} \right] \mathrm{e}^{\imath k\lambda},$$

where $\omega > 0$ and $\lambda > 0$ is a real parameter. For an analytical function f satisfying, in the whole complex plane, the condition $|f(k)|\,\mathrm{e}^{-\lambda|\mathrm{Im}[k]|} \to 0$, for $|\mathrm{Im}[k]| \to \infty$, we get $\mathcal{A}_\lambda^{(\pm)}(\omega, f) = \mp\imath\pi f(\omega)\,\mathrm{e}^{\imath\omega\lambda} + \int_0^\infty \frac{\mathrm{d}k}{k+\omega}\left[f(k)\,\mathrm{e}^{\imath k\lambda} \mp f(-k)\,\mathrm{e}^{-\imath k\lambda}\right]$. Hence, the (rr) contributions to the potential for each polarizations are given by

$$V^{rr}_{a,z}(\rho,\phi,q) = \frac{1}{4}\sum_b \hbar\Gamma^z_{a\to b}\left\{\sum_{l=0}^{q-1} U^{rr}_\parallel(2|k_{ba}|\rho\sin(\phi+\pi l/q)) - \sum_{l=1}^{q-1} U^{rr}_\parallel(2|k_{ba}|\rho\sin(\pi l/q))\right\},$$

$$V^{rr}_{a,\phi}(\rho,\phi,q) = \frac{1}{4}\sum_b \hbar\Gamma^\phi_{a\to b}\left\{\sum_{l=0}^{q-1} W^\phi_{rr}(2|k_{ba}|\rho,\phi+\pi l/q) + \sum_{l=1}^{q-1} W^\phi_{rr}(2|k_{ba}|\rho,\pi l/q)\right\},$$

$$V^{rr}_{a,\rho}(\rho,\phi,q) = \frac{1}{4}\sum_b \hbar\Gamma^\rho_{a\to b}\left\{\sum_{l=0}^{q-1} W^\rho_{rr}(2|k_{ba}|\rho,\phi+\pi l/q) - \sum_{l=1}^{q-1} W^\rho_{rr}(2|k_{ba}|\rho,\pi l/q)\right\},$$

where $\Gamma^\sigma_{a\to b} = \frac{4}{\hbar}|\langle a|d_\sigma|b\rangle|^2|k_{ba}|^3$, and the functions in last equations are defined as

$$U^{rr}_\perp(x) = -\frac{1}{2x^3}(\cos x + x\sin x); \qquad U^{rr}_\parallel(x) = -\frac{1}{2x^3}(\cos x + x\sin x - x^2\cos x),$$
$$W^\phi_{rr}(x,\psi) = U^{rr}_\parallel(x\sin\psi)\sin^2\psi + 2U^{rr}_\perp(x\sin\psi)\cos^2\psi,$$
$$W^\rho_{rr}(x,\psi) = U^{rr}_\parallel(x\sin\psi)\cos^2\psi + 2U^{rr}_\perp(x\sin\psi)\sin^2\psi.$$

The (fr) contribution to the level shift, δE^{fr}_a, can be obtained by a completely analogous procedure. This contribution depends on the state of the field. Here, we consider the field in the vacuum state, $\langle n_{\mathbf{k}\lambda}\rangle = 0$. In this particular case, it can be shown that the (fr) dispersion potential, $V^{fr}_{a,\sigma}(\rho,\phi)$, may be obtained from previous equations by the shortcut:

$$U^{rr}_{\parallel(\perp)}(x) \longrightarrow U^{fr}_{\parallel(\perp)}(x) = U_{\parallel(\perp)}(x) - U^{rr}_{\parallel(\perp)}(x), \qquad \sum_b \longrightarrow \sum_{b>a} - \sum_{b<a},$$

where the functions $U_\perp$ and $U_\parallel$ are defined by (γ is the Euler–Mascheronni constant)

$$U_\perp(x) = \frac{1}{\pi x^3}\left[-\mathcal{F}(x) + x\mathcal{G}(x)\right]; \qquad U_\parallel(x) = \frac{1}{\pi x^3}[(x^2-1)\mathcal{F}(x) + x\mathcal{G}(x) - x],$$
$$\mathcal{F}(x) = \int_0^\infty \mathrm{d}t\,\frac{\sin t}{t+x} = \mathrm{C}i(x)\sin x - \mathrm{s}i(x)\cos x,$$
$$\mathcal{G}(x) = -\int_0^\infty \mathrm{d}t\,\frac{\cos t}{t+x} = \mathrm{C}i(x)\cos x + \mathrm{s}i(x)\sin x = \frac{\mathrm{d}}{\mathrm{d}x}\mathcal{F}(x),$$
$$\mathrm{s}i(x) = -\frac{\pi}{2} + \int_0^x \mathrm{d}t\,\frac{\sin t}{t}, \mathrm{C}i(x) = \gamma + \ln x + \int_0^x \mathrm{d}t\,\frac{\cos t - 1}{t}.$$

The full dispersive potential of an atom in a state $|a\rangle$ is given by the sum of both (rr) and (fr) contributions, so that $V_a(\rho,\phi,q) = \sum_\sigma \left(V^{rr}_{a,\sigma}(\rho,\phi,q) + V^{fr}_{a,\sigma}(\rho,\phi,q)\right)$. Let us, then, compute the van der Waals potential between the atom and the wedge. For

this case, the atom is in its ground state ($a \to g$), and assuming spherical symmetry, $|\langle g|d_z|e\rangle|^2 = |\langle g|d_\phi|e\rangle|^2 = |\langle g|d_\rho|e\rangle|^2 = \frac{1}{3}|\langle g|\mathbf{d}|e\rangle|^2$, where the index e specifies an excited state, we finally obtain

$$V_g(\rho,\phi,q) = \frac{1}{2}\sum_e \hbar\Gamma^{\rm spt}_{e\to g}\left\{\sum_{l=0}^{q-1} U(2k_{eg}\rho\sin(\phi+\pi l/q)) - \sum_{l=1}^{q-1}[U_\parallel(2k_{eg}\rho\sin(\pi l/q)) \times \cos^2(\pi l/q) - U_\perp(2k_{eg}\rho\sin(\pi l/q))\cos(2\pi l/q)]\right\}, \tag{5}$$

where $U(x) = U_\parallel(x) + U_\perp(x)$ and $\Gamma^{\rm spt}_{e\to g} = \frac{4}{3\hbar}|\langle g|\mathbf{d}|e\rangle|^2 k_{eg}^3$. This is the main result of the present paper. As a check of our result, let us reobtain from the above equation a few known results existent in the literature. Let us start with the atom–wall interaction. This case corresponds to take $\phi_0 = \pi$ ($q = 1$). For any distance regime, we get

$$V_g(\rho,\phi,q=1) = \frac{1}{2}\sum_e \hbar\Gamma^{\rm spt}_{e\to g} U(2k_{eg}z)\ (z := \rho\sin\phi), \tag{6}$$

which was shown in [23] to yield the well-known results in the literature. Further, in order to particularize the above result for the non-retarded and retarded regimes we must take the appropriate approximations of the functions $U_\perp$ and $U_\parallel$. For the former (short distances) we have $U_\perp(x) \simeq U_\parallel(x) \simeq -\frac{1}{2x^3}$, while for the latter (large distances) we have $U_\parallel(x) \simeq 2U_\perp(x) \simeq -\frac{4}{\pi x^4}$. Substituting into (5) the short-distance behaviour of these functions as well as the relation $\Gamma^{\rm spt}_{e\to g} = \frac{4}{3\hbar}|\langle g|\mathbf{d}|e\rangle|^2 k_{eg}^3$, the non-retarded atom–wall interaction is given by

$$V_g^{\rm NR}(z) = -\frac{1}{12z^3}\sum_e |\langle g|\mathbf{d}|e\rangle|^2. \tag{7}$$

Since we assumed spherical symmetry, we may write $\alpha^\rho_{ge} = \alpha^\phi_{ge} = \alpha^z_{ge} =: \alpha_{ge}$, and from the definition of α^j_{ab} given after equation (4), we have $\alpha_{ge} = \frac{2}{3}|\langle g|\mathbf{d}|e\rangle|^2/(\hbar c k_{eg})$ (recall that $k_{ge} = -k_{eg}$). Hence, equation (7) can be written in terms of α_{ge} as

$$V_g^{\rm NR}(z) = -\frac{\hbar c}{8z^3}\sum_e \alpha_{ge}k_{eg}. \tag{8}$$

Substituting now into (6) the large-distance behaviour of the functions $U_\perp$ and $U_\parallel$, we get

$$V_g^R(z) = -\frac{3\hbar c}{8\pi z^4}\sum_e \alpha_{ge} = -\frac{3\hbar c\alpha(0)}{8\pi z^4}, \tag{9}$$

where $\alpha(0)$ is the static polarizability of the atom.

For an atom in the region between two parallel conducting plates, discussed in detail by Barton [7], we were not able to establish an equivalence analytically. However, we have checked numerically that both results are in complete agreement.

Now, as our last particular case, we reobtain the retarded potential for the atom–wedge system calculated by Brevik *et al* [19]. For large distances, our general result (5) takes the form (this is called the Casimir–Polder regime)

$$V_{\rm CP}(\rho,\phi,q) = \frac{\hbar}{32\pi\rho^4}\sum_e \frac{\Gamma^{\rm spt}_{e\to g}}{|k_{eg}|^4}\left[-6\sum_{l=0}^{q-1}\sin^{-4}(\phi+\pi l/q) + 2\sum_{l=1}^{q-1}\sin^{-4}(\pi l/q)\right].$$

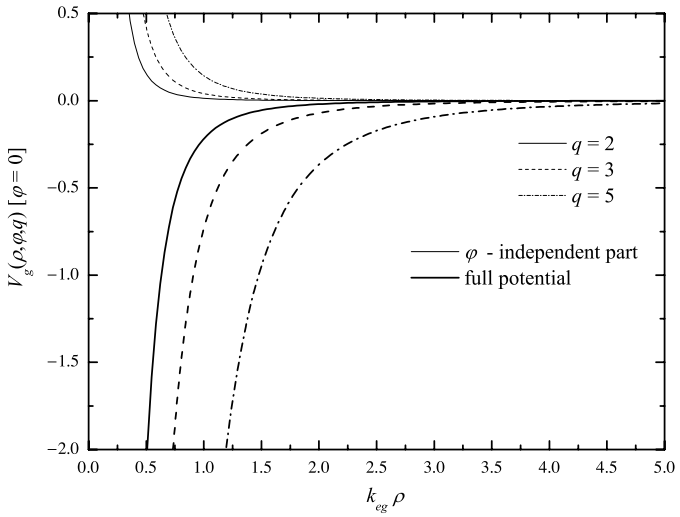

Figure 2. $V_g(\rho, \varphi = 0, q)$ versus ρ for different values of q. For increasing q (closing the wedge) the interaction becomes stronger, as expected.

Previous summations can be evaluated and the previous result can be cast into the form

$$V_{\rm CP}(\rho, \phi, q) = \frac{\hbar c \alpha(0)}{4\pi\rho^4}\left[\frac{1}{90}(q^2-1)(q^2+11) - \frac{q^2}{\sin^2 q\phi}\left(\frac{3q^2}{2\sin^2 q\phi} + 1 - q^2\right)\right],$$

considered now valid for every value of q, in perfect agreement with [19].

Figure 2 shows the variation of $V(\rho, \phi, q)$, given by equation (5), in terms of ρ for fixed ϕ, taken conveniently as $\phi = \phi_0/2$ ($\varphi = 0$) and for different values of q. The full potential is represented by the strong lines: the solid one corresponding to $\phi_0 = \pi/2$, the dashed one to $\phi_0 = \pi/3$ and the dotted-dashed one to $\phi_0 = \pi/5$. The component of the force along $\hat{\rho}$, denoted by F_ρ, is attractive. However, looking at equation (6), we note that the last two contributions are independent of ϕ and with a repulsive character. In other words, no matter the value of ϕ, there is always a repulsive corner contribution to the force F_ρ, represented in figure 2 by the thin lines.

Figure 3 shows $F_\rho = -\frac{\partial V_g}{\partial \rho}$ in terms of ϕ for fixed ρ and for different values of q. Note that the modulus of F_ρ increases as we get closer and closer to each plate. Even for $\phi = \phi_0/2$ ($\varphi = 0$), i.e., for points equidistant from both plates forming the wedge, F_ρ is not zero, though it assumes minimum values at these points. Observe, also, that for $\varphi = 0$ the modulus of F_ρ increases as the angle between the plates diminishes. Naively, one could think this is a paradoxal result, since the plates are becoming more and more parallel to each other and for an atom inside two parallel plates there is no component of the force parallel to the plates. However, to obtain correctly the limit of an atom inside two parallel plates one must not only diminish the angle between the plates but also take ρ to infinite, with the constraint $\rho\phi_0 = a$, a being the distance between the parallel plates. A simple but careful analysis shows that, in fact, $F_\rho(\rho, \varphi = 0, q)$ increases if we maintain ρ fixed and finite and increase q (close to the wedge).

Figure 4 shows $F_\phi = -\frac{1}{\rho}\frac{\partial V_g}{\partial \phi}$ in terms of ϕ for fixed ρ and for different values of q. In contrast to what happens to F_ρ, $F_\phi = 0$ for $\varphi = 0$. Besides, $F_\phi < 0$ for negative φ and $F_\phi > 0$

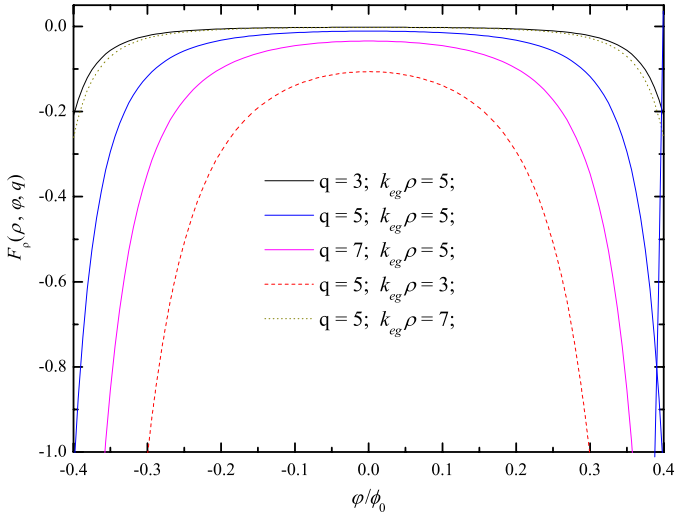

Figure 3. $F_\rho(\rho, \varphi, q)$ with fixed ρ versus φ/ϕ_0 for different values of q and ρ.

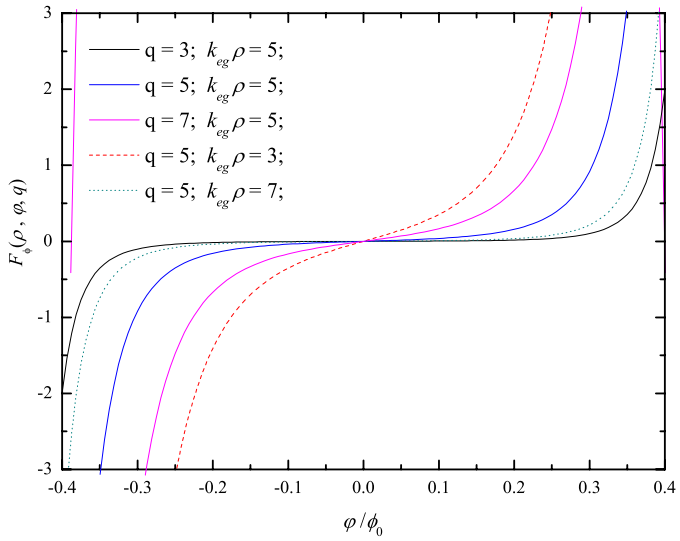

Figure 4. $F_\phi(\rho, \varphi, q)$ with fixed ρ versus φ/ϕ_0 for different values of q and ρ.

for positive φ: except for $\varphi = 0$, the atom is attracted to the nearest plate (apart from a F_ρ component pointing to the corner).

3. Final comments and perspectives

Using an approach based on a master equation, we computed the dispersion force on an electrically polarizable atom near a perfectly conducting wedge. Our result is valid for both retarded and non-retarded regimes, generalizing in this way a previous result of the literature

[19], which is valid only in the retarded regime. We checked our calculations by reobtaining well-known results in the literature, as for instance, the non-retarded and retarded interactions between an atom and a perfectly conducting wall, given by equations (7) (or equivalently (8)) and (9). It is worth noting that since the non-relativistic expression $|\langle g|\mathbf{d}|e\rangle|^2$ does not depend on c, the non-retarded potential is independent of c (see equation (7)), as it should be (the dependence on $\hbar$ comes from $|\langle g|\mathbf{d}|e\rangle|^2$). In (8), the dependence on c is just apparent, since it cancels with that appearing in $k_{eg} = \omega_{eg}/c$. We should emphasize, however, that in the non-retarded regime expression (5) is valid only for $\phi_0 = \pi/q$, with q a positive integer number and ϕ_0 being the angle of the wedge. We found a full ρ-component of the force which is attractive. There is, however, a ϕ-independent repulsive contribution for F_ρ, which we interpreted as a corner contribution. A numerical analysis should be made to see if our results can be of some relevance in future experiments using this geometry. The method employed here can be applied in the computation of resonant potentials (excited atoms), but this will be left for a future work. Our formalism can still be used in the computation of spontaneous emission rates of an atom near boundaries as the one considered here.

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